$\bar{X}$ - and S - Charts.

For each sample i, $\quad S_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(x_{ij}-\bar{x})^2}$ can be calculated

In general $Var[S] = E[S^2] - (E[S])^2 = \sigma^2 - (E[S])^2 > 0$

$\Rightarrow E[S] < \sigma$

$$E[S] = c_4\sigma \quad \text{where} \quad c_4 = \left(\frac{2}{n-1}\right)^{\frac{1}{2}} \frac{\Gamma(\frac{n}{2})}{\Gamma(\frac{n-1}{2})}$$

Hence $Var[S] = \sigma^2(1-c_4^2)$ and a natural chart would be

$$UCL = c_4\sigma + 3\sigma\sqrt{1-c_4^2}, \quad CL = c_4\sigma, \quad LCL = c_4\sigma - 3\sigma\sqrt{1-c_4^2}$$

if the variance was known.

If not an overall estimate for σ is given by.

$$\frac{\bar{S}}{c_4} = \frac{\left(\frac{1}{k}\sum_{i=1}^{k} S_i\right)}{c_4}$$

which gives the chart.

$$UCL \quad \bar{S} + \frac{3\bar{S}}{c_4}\sqrt{1-c_4^2}, \quad CL = \bar{S}, \quad LCL = \bar{S} - \frac{3\bar{S}}{c_4}\sqrt{1-c_4^2}$$

or. $\bar{S}\left(1 + \frac{3}{c_4}\sqrt{1-c_4^2}\right), \quad \bar{S}, \quad \bar{S}\left(1 - \frac{3}{c_4}\sqrt{1-c_4^2}\right)$

i.e. $UCL = \bar{S}B_4, \quad CL = \bar{S}, \quad LCL = \bar{S}B_3$

where $B_4 = 1 + \frac{3}{c_4}\sqrt{1-c_4^2}, \quad B_3 = 1 - \frac{3}{c_4}\sqrt{1-c_4^2}.$ $\quad B_4$ an B_3 from Table A22.

The $\bar{x}$ - chart is now.

$$UCL: \bar{\bar{x}} + 3\frac{\hat{\sigma}}{\sqrt{n}}, \quad CL = \bar{\bar{x}}, \quad LCL = \bar{\bar{x}} - 3\frac{\hat{\sigma}}{\sqrt{n}}$$

which with $\hat{\sigma} = \frac{\bar{s}}{c_4}$ becomes

$$UCL = \bar{\bar{x}} + A_3\bar{s}, \quad CL = \bar{\bar{x}}, \quad LCL = \bar{\bar{x}} - A_3\bar{s}$$

where $A_3 = \frac{3}{c_4\sqrt{n}}$, Table A22.

17.5. Controll Charts for Attributes

p- chart.

Let X be the number of defective items in a sample of size n.

Under the assumptions of independence and equal probability, p, of being defective,

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x} \quad \text{i.e } X \sim \text{binomial distributed } (n, p)$$

and $\hat{p} = \frac{X}{n}$. $\quad E[\hat{p}] = \frac{E[X]}{n} = \frac{np}{n} = p.$

$$Var(\hat{p}) = \frac{Var[X]}{n} = \frac{np(1-p)}{n^2} = \frac{p(1-p)}{n}$$

If. p is known the control chart becomes.

$$LCL = p - 3\sqrt{\frac{p(1-p)}{n}}, \quad CL = p, \quad UCL = p + 3\sqrt{\frac{p(1-p)}{n}}$$

Suppose p is unknown. From m samples p can estimated from $\bar{p} = \frac{1}{m}\sum_{i=1}^{m} \hat{p}_i$

and the control limits become

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}, \quad CL = \bar{p}, \quad UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

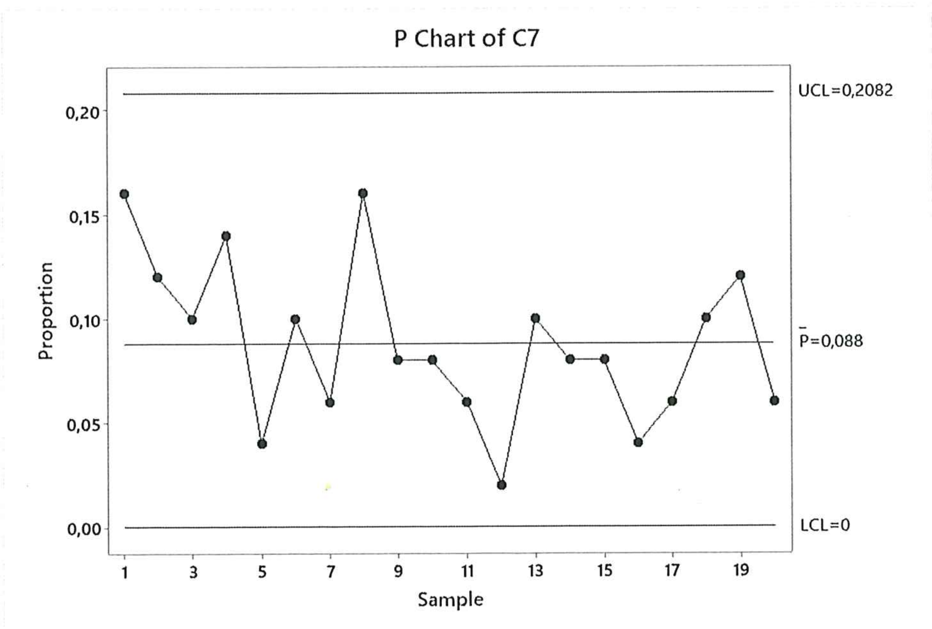
How to decide on n. One possibility is to choose n such that there is a probability 0.5 of detecting a shift of size $p_1 - p$ where $p_1 > p$. Assume $\frac{X}{n} \approx N\left(p_1, \frac{p_1(1-p_1)}{n}\right)$

$$P(\hat{p} \geq UCL) \approx P\left[\frac{\hat{p} - p_1}{\sqrt{\frac{p_1(1-p_1)}{n}}} \geq \frac{UCL - p_1}{\sqrt{\frac{p_1(1-p_1)}{n}}}\right] = 0.5$$

Defective Electronical Components

Data: Number of defective out of 50

Row	C7
1	8
2	6
3	5
4	7
5	2
6	5
7	3
8	8
9	4
10	4
11	3
12	1
13	5
14	4
15	4
16	2
17	3
18	5
19	6
20	3



$$\Rightarrow \frac{UCL - p_1}{\sqrt{\frac{p_1(1-p_1)}{n}}} = 0 \Rightarrow \frac{p + 3\sqrt{\frac{p(1-p)}{n}} - p_1}{\sqrt{\frac{p_1(1-p_1)}{n}}} = 0$$

$$\Rightarrow (p - p_1) + 3\sqrt{\frac{p(1-p)}{n}} = 0$$

or $3\sqrt{\frac{p(1-p)}{n}} = p_1 - p = \Delta$ .. We get,

$$9\,\frac{p(1-p)}{n} = \Delta^2 \Rightarrow n = \frac{9p(1-p)}{\Delta^2}.$$

C- charts

Let X ~~be~~ be the number of defects or number of non conformities on a product.

It is reasonable that $P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$, $x = 0, 1, 2, \dots$

$E[X] = \lambda$, $Var[X] = \lambda$ and a reasonable chart is

$$LCL = \lambda - 3\sqrt{\lambda}, \quad CL = \lambda, \quad UCL = \lambda + 3\sqrt{\lambda}$$

If X_i is the number of defects in sample i, $i = 1, 2, \dots, m$

$\hat{\lambda} = \frac{1}{m}\sum_{i=1}^{m} X_i$ which gives the chart,

$$LCL = \hat{\lambda} - 3\sqrt{\hat{\lambda}}, \quad CL = \hat{\lambda}, \quad UCL = \hat{\lambda} + 3\sqrt{\hat{\lambda}}$$

U- charts

The subsample has n products. Let $U = \sum_{i=1}^{n} X_i$. $E[U] = n\lambda$ and $Var[U] = n\lambda \Rightarrow E[\frac{U}{n}] = \lambda$ and $Var[\frac{U}{n}] = \frac{\lambda}{n}$

Let $\bar{U} = \frac{1}{m}\sum_{i=1}^{m} X_i \Rightarrow$

$$LCL = \bar{U} - 3\sqrt{\frac{\bar{U}}{n}}, \quad CL = \bar{U}, \quad UCL = \bar{U} + 3\sqrt{\frac{\bar{U}}{n}}$$

Number of accidents in industry

5 5 10 8 4 5 7 3 2 8 6 9 5 6 5 10 6 3 3 10 3 4 2 0 1 3 2 2 7 7
1 4 1 2 2 1 4 4 4 4